Benchmarking New Magic Tricks

PsiQ USYD FT Meeting ‘26
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Outline

1. Overview
2. Twisted Quantum Double
3. Simulation Procedure

Overview

Overview

Algorithms require universal gatesets.

Eg. {H, CX, S, T}, more generally {Cliffords, any non-Clifford gate}

[Eastin-Knill] No QECC with distance > 1 supports a universal gate set with all transversal implementation.

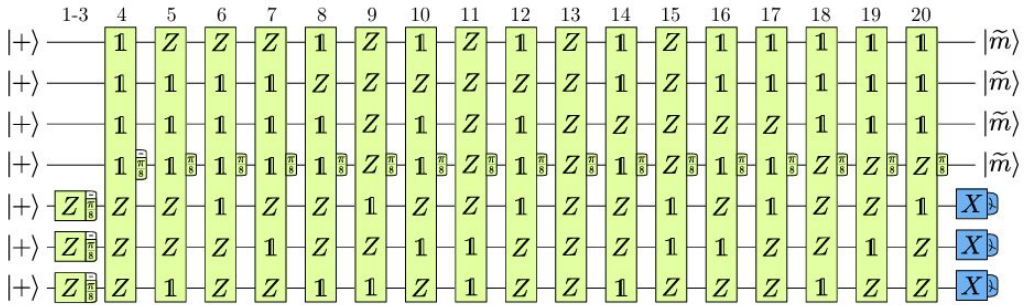
Clifford gates map Pauli errors to Pauli errors, but non-Cliffords could map them to Clifford errors!

[Bravyi-König] For topological QECCs in 2D, logical gates implemented by locality-preserving gates are restricted to being Cliffords.

Question: How do we apply non-Clifford gates fault-tolerantly?

Overview

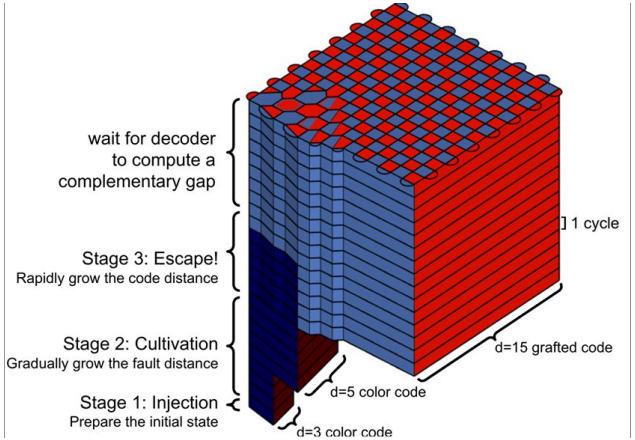
- Magic State Distillation: Many (impure) → Few (high fidelity)
  Fault Tolerant, Uses Clifford operations
  High Spacetime cost



20-to-4 MSD Circuit ([2] Litinski)

Overview

- Magic State Distillation: High Spacetime cost
- Magic State Cultivation: Lower Spacetime cost, Does not scale well



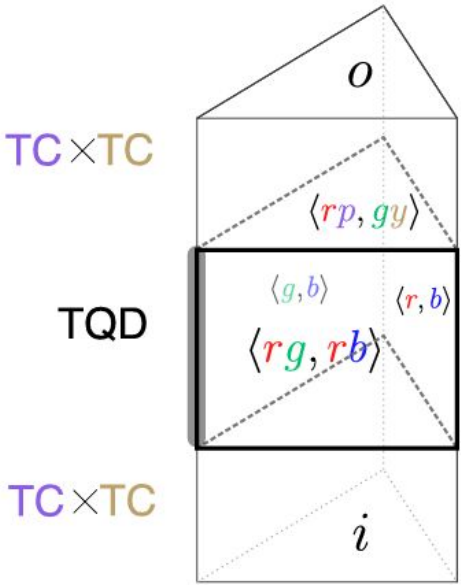
Overview of Cultivation Process (Fig. 4, Gidney et. al. [3])

Overview

- Magic State Distillation: Expensive
- Magic State Cultivation: Not scalable

- New Gauging approach:

Logical T State Preparation using a Color Code ([1] Davydova et. al.)

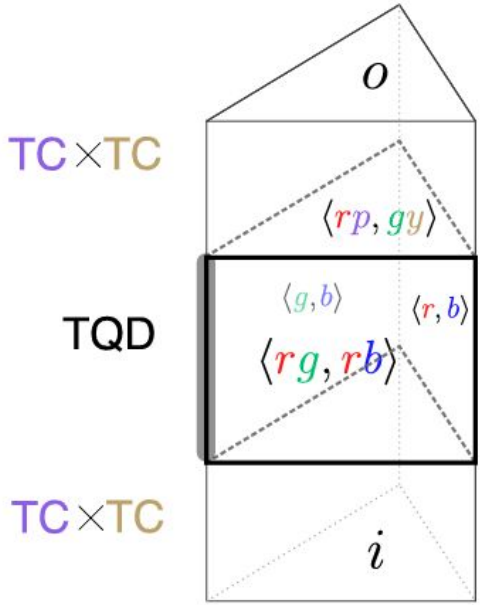


Overview

- New Gauging approach:

Goal: Prepare $|\overline{T}\rangle$ state

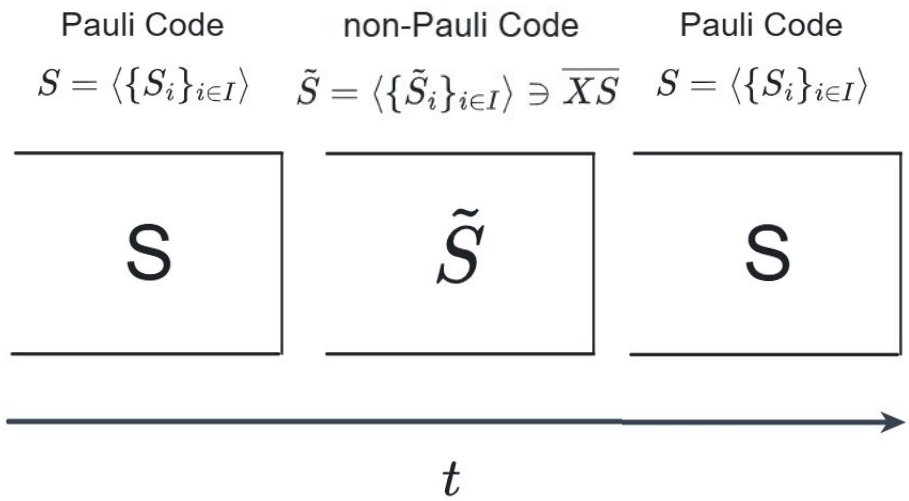
- Want $|\overline{T}\rangle$ state, which is +1 eigenstate of $\overline{X}\overline{S}$
- Idea: Project onto eigenstates of $\overline{X}\overline{S}$ by adding it to stabilizer group



Logical T State Preparation using a Color Code ([1] Davydova et. al.)

Protocol

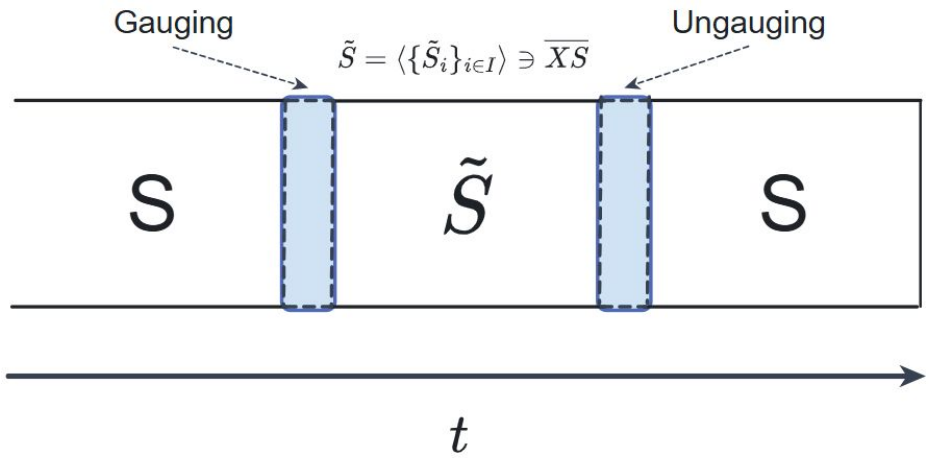
- Want $|\overline{T}\rangle$ state, which is +1 eigenstate of $\overline{X}\overline{S}$
- Idea: Project onto eigenstates of $\overline{X}\overline{S}$ by adding it to stabilizer group



- non-Pauli code = some stabilizers are Clifford gates; only necessarily commute on codespace i.e. shared +1 eigenspace

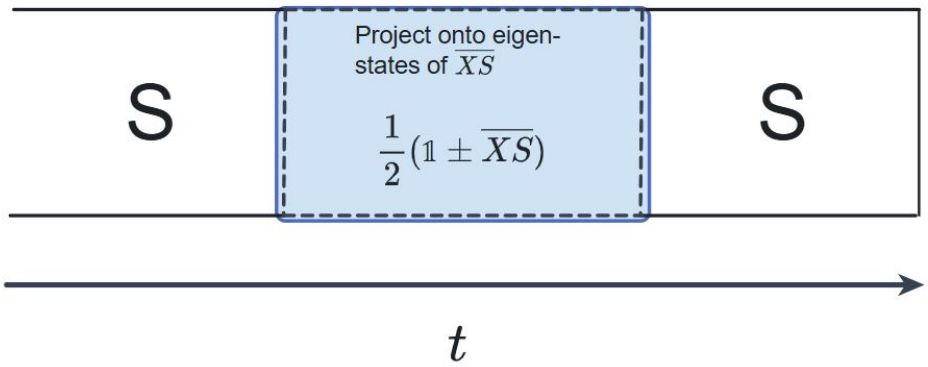
Protocol

- Want $|\overline{T}\rangle$ state, which is +1 eigenstate of $\overline{X}\overline{S}$
- Idea: Project onto eigenstates of $\overline{X}\overline{S}$ by adding it to stabilizer group



Protocol

- Want $|\overline{T}\rangle$ state, which is +1 eigenstate of $\overline{X}\overline{S}$ (up to phase)
- Idea: Project onto eigenstates of $\overline{X}\overline{S}$ by adding it to stabilizer group



Protocol

- Logical Information transferred at domain walls, boundary conditions must match

Twisted Quantum Double Model
(non-Abelian phase)



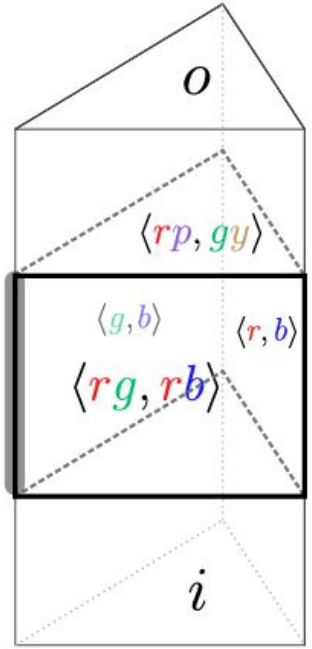
TQD

Two copies of TC joined with
“fold boundary”, equivalent to
2D Color Code with Triangular BC

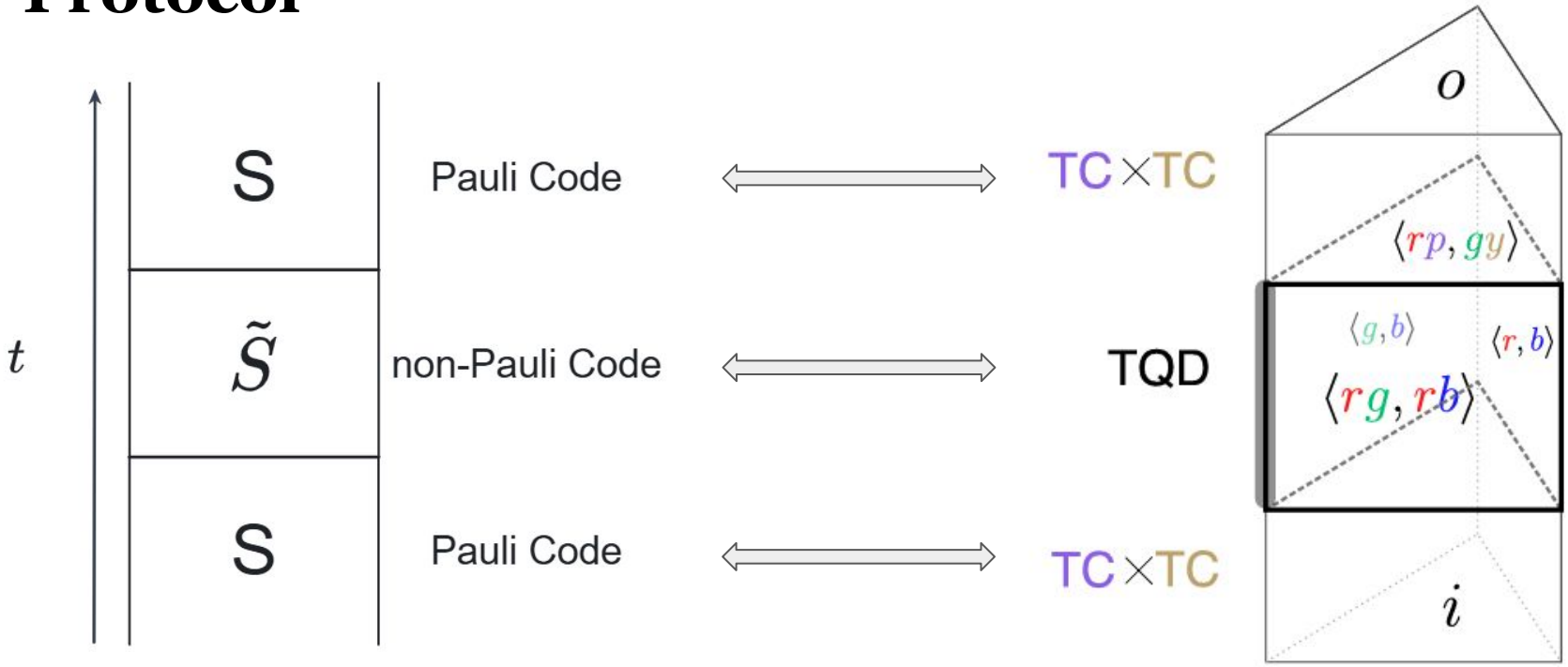


TC×TC

TC×TC



Protocol


t
S
$\tilde{S}$
S
Pauli Code
non-Pauli Code
Pauli Code
TC×TC
TQD
TC×TC
o
$\langle rp, gy \rangle$
$\langle g, b \rangle$
$\langle r, b \rangle$
$\langle rg, rb \rangle$
i

Overview

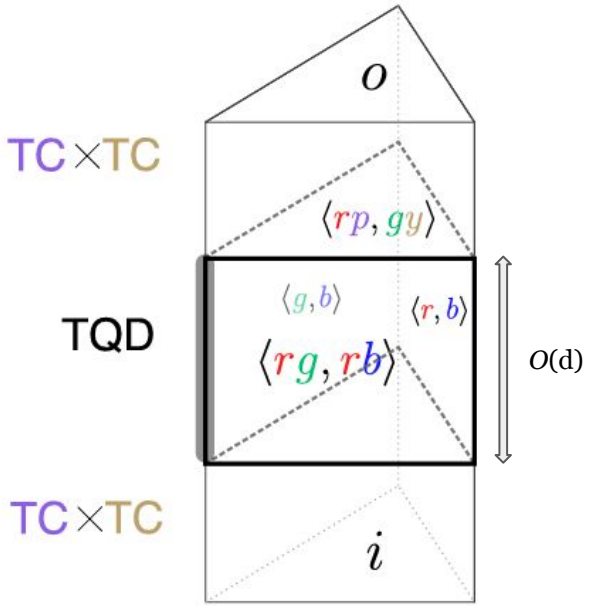
- New Gauging approach:

  Fault Tolerant, Scalable

  Proven threshold theorem

  Can prepare various non-Clifford Logical Gates

Question: How practical is this approach?



Logical T State Preparation using a Color Code ([1] Davydova et. al.)

Overview

2503.15751 (PRX 16, 031001) = M. Davydova et. al. Universal Fault Tolerant Quantum Computation in 2D without getting tied in knots

→ T State Preparation protocol

2604.02033 = J. Magdalena de la Fuente et. al. High-threshold decoding of non-Pauli codes for 2D universality

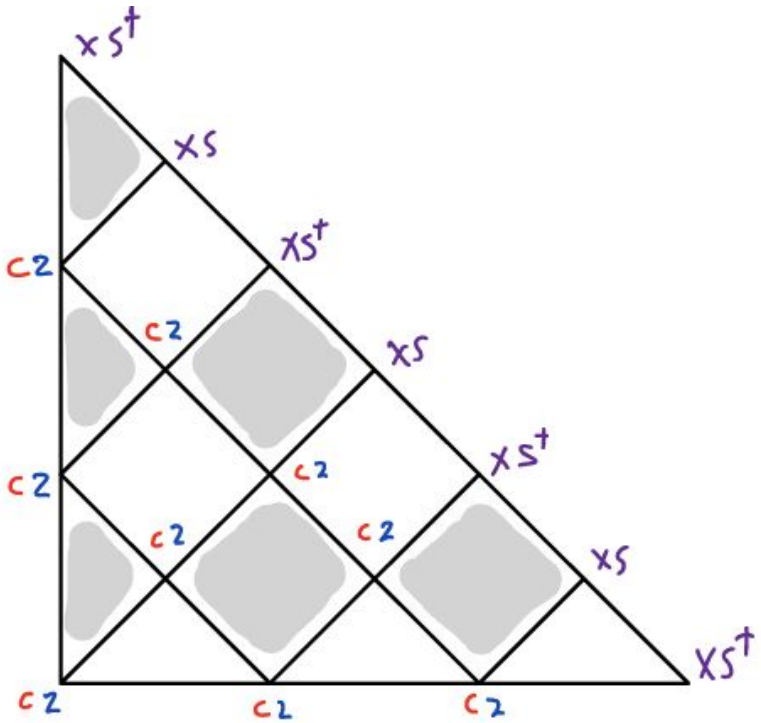
→ Just-in-Time decoder

2609.1629 = Y. Takada et. al. Exact efficient simulation of noisy logical magic states using Clifford stabilizers

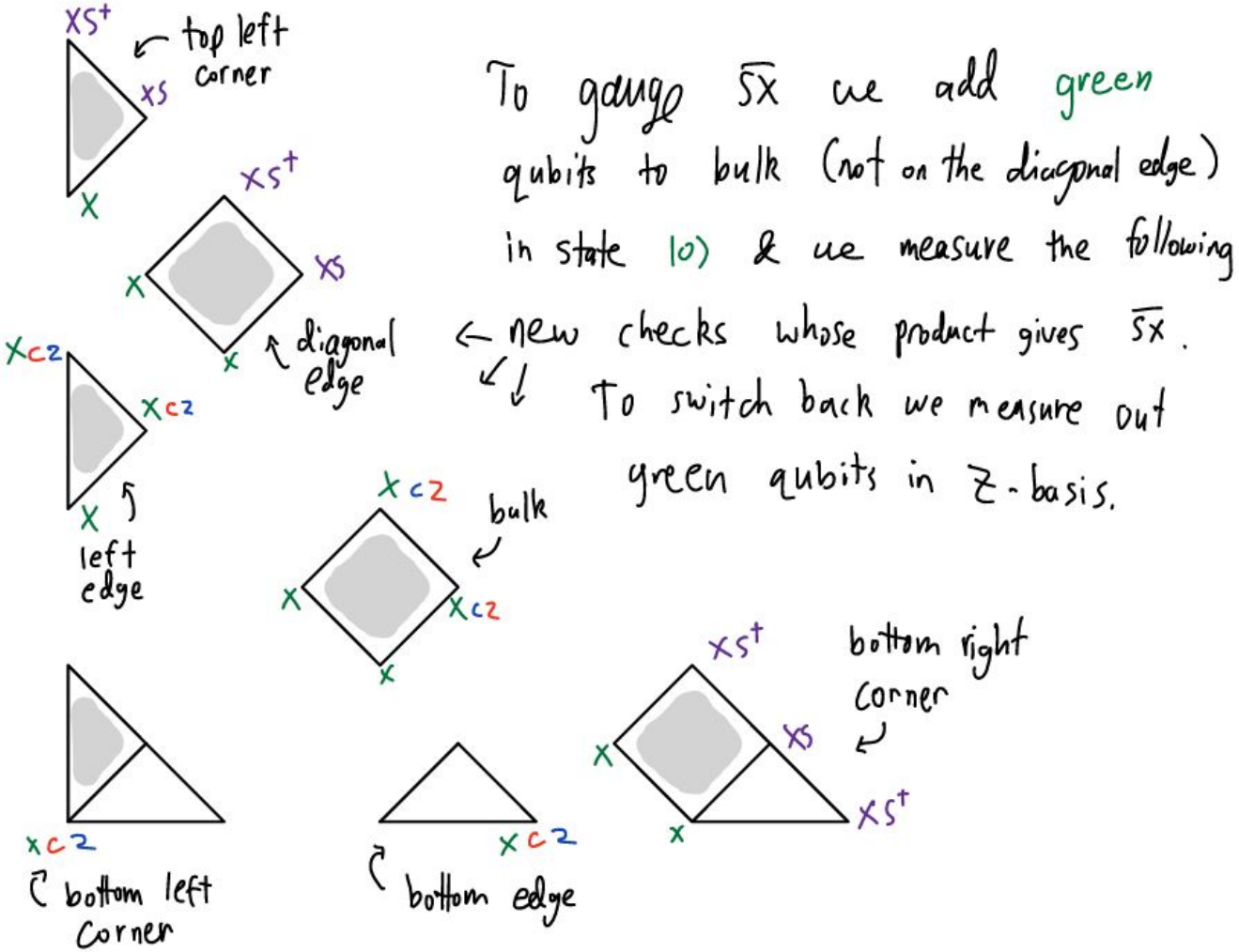
→ Measuring TQD stabilizers (requires physical non-clifford gates like CCZ, CXS, C-$XS^{\dagger}$)



Actual TQD we're using



- Has transversal logical XS that we can gauge by adding green qubits to thee "bulk"



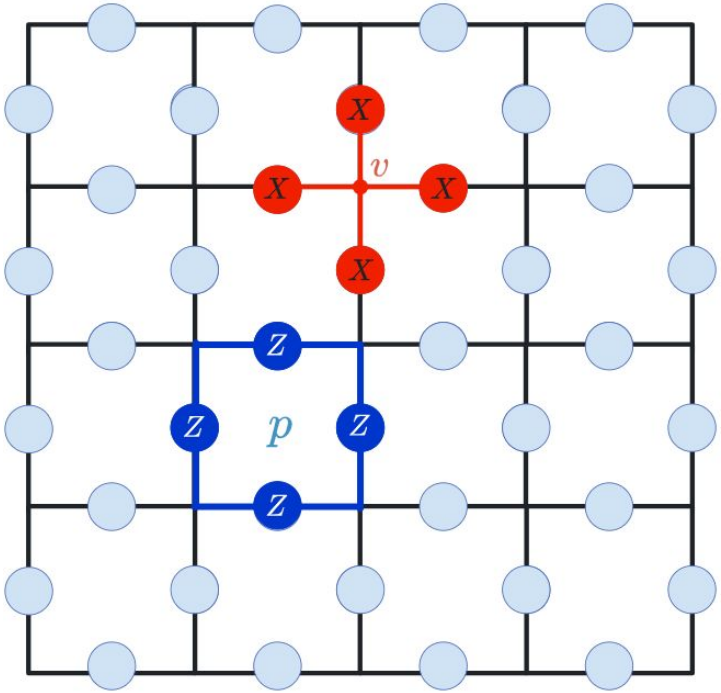
To gauge $\bar{S}\bar{X}$ we add green qubits to bulk (not on the diagonal edge) in state $|0\rangle$ & we measure the following new checks whose product gives $\bar{S}\bar{X}$.

To switch back we measure out green qubits in Z-basis.

Twisted Quantum Double

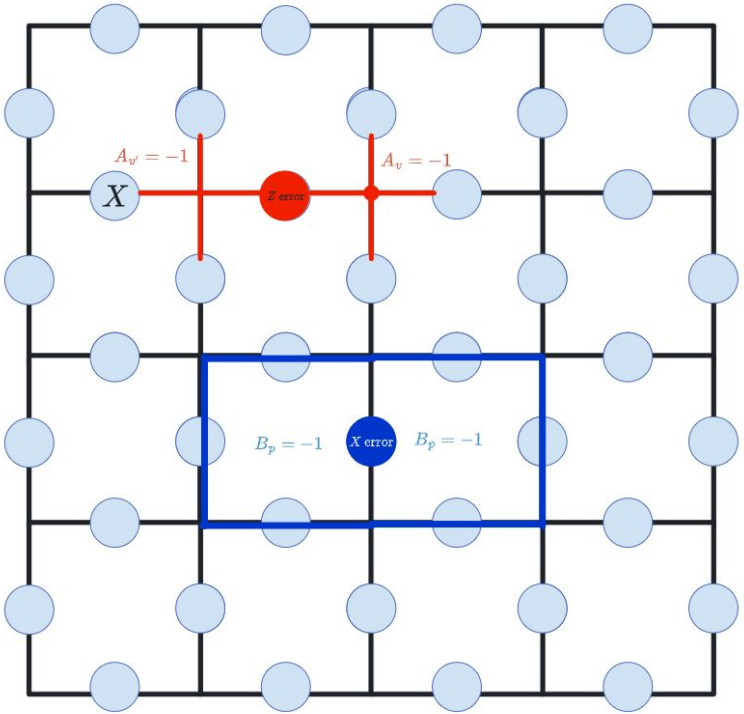
Super quick Surface Code review

- Vertex and Plaquette Stabilizers

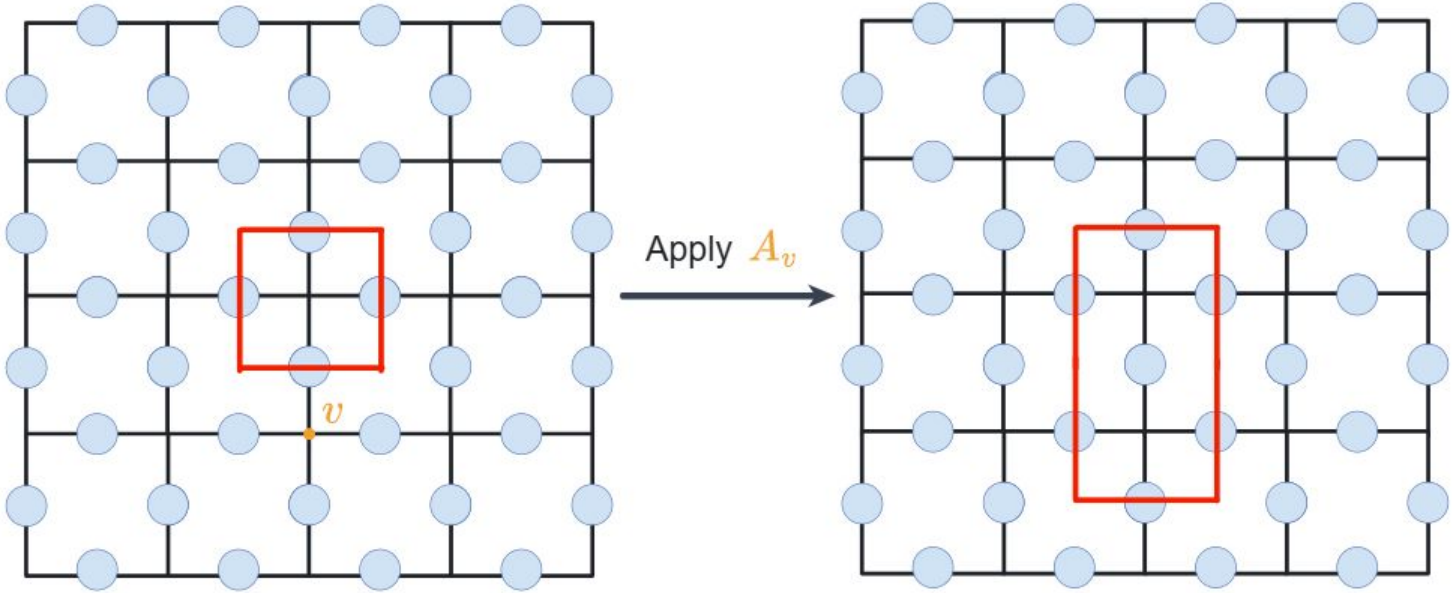


Super quick Surface Code review

- Vertex and Plaquette Stabilizers, e and m anyons ↔ Stab. Violations



Super quick Surface Code review



- B_p enforces ground states are loop configs.
- A_v maps one loop config to another.

Super quick Surface Code review

- Equivalences and Constraints on charge (c), flux (b) configurations for TC ground state:

Fluxes:

$$db \neq 0 \quad \Rightarrow \quad C_{c,b} = 0 \,,$$
$$C_{b+d\beta,c} \propto C_{b,c} \quad \forall \beta \in C^1 \,.$$

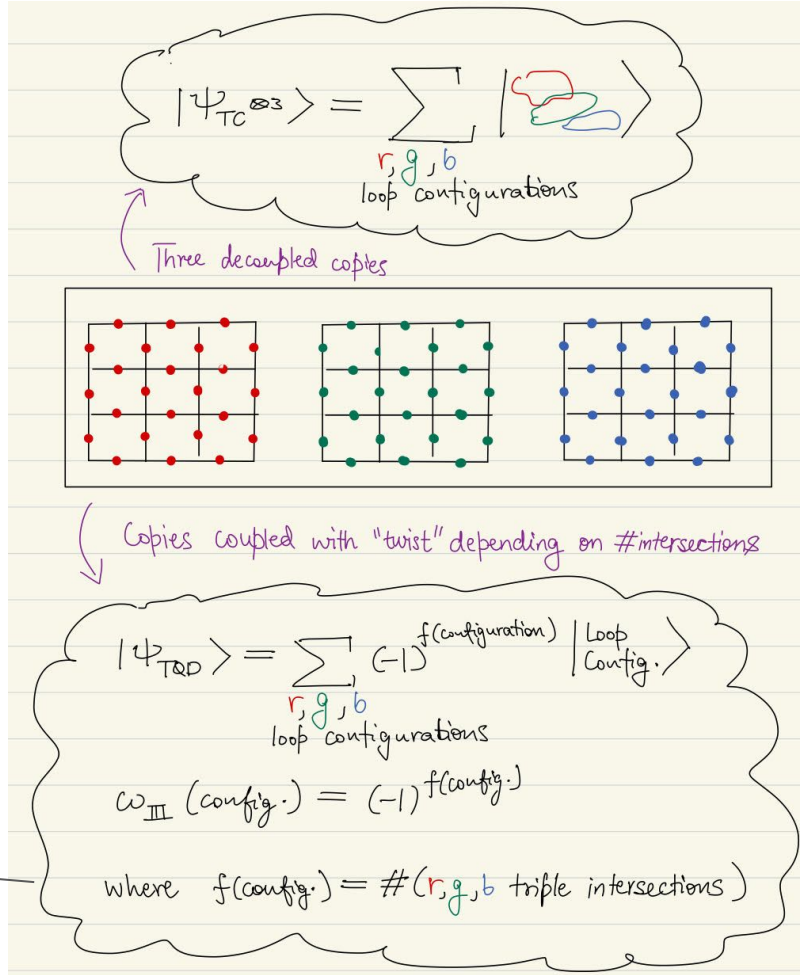
Charges:

$$d^T c \neq 0 \quad \Rightarrow \quad C_{c,b} = 0 \,,$$
$$C_{b,c+d^T\gamma} \propto C_{b,c} \quad \forall \gamma \in C^2 \,.$$

[4]. J. M. de la Fuente et. al.

Twisted Quantum Double

- Three Surface Codes coupled together with a “Twist”



Eg. For $|$  $\rangle$, $f(|$  $\rangle) = 0$

But, $f(|$  $\rangle) = 1$

Twisted Quantum Double

- Stabilizers only necessarily commute on +1 eigenspace.

- (Twisted) Vertex / Plaquette Stabilizer violations are resp. Abelian / non-Abelian
- Problem: Some stabilizer violations can lead to/hide other violations
  → Non-Abelian Anyons/Defects

$$[A_{v,c}, A_{v',c'}]_G \in \langle \{B_{f,c}\}_{f,c} \rangle$$

Twisted Quantum Double

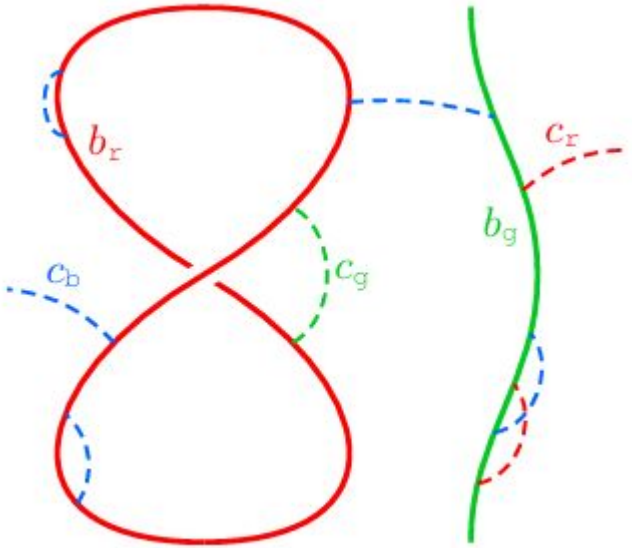
[4]. J. M. de la Fuente et. al.

$$[A_{v,c}, A_{v',c'}]_G \in \langle \{B_{f,c}\}_{f,c} \rangle$$

TQD Equivalences and Constraints:

$$db \neq 0 \quad \Rightarrow \quad C_{c,b} = 0 \,,$$
$$C_{b+d\beta,c} \propto C_{b,c} \quad \forall \beta \in C^1 : \beta \text{ isolated from } b \,,$$

$$K^b d^T c \neq \kappa^b \quad \Rightarrow \quad C_{c,b} = 0 \,,$$
$$C_{b,c+d^T\gamma+\widetilde{M}^b v} \propto C_{b,c} \quad \forall \gamma \in C^2, v \in C^0 \,.$$



Just-in-time Decoder

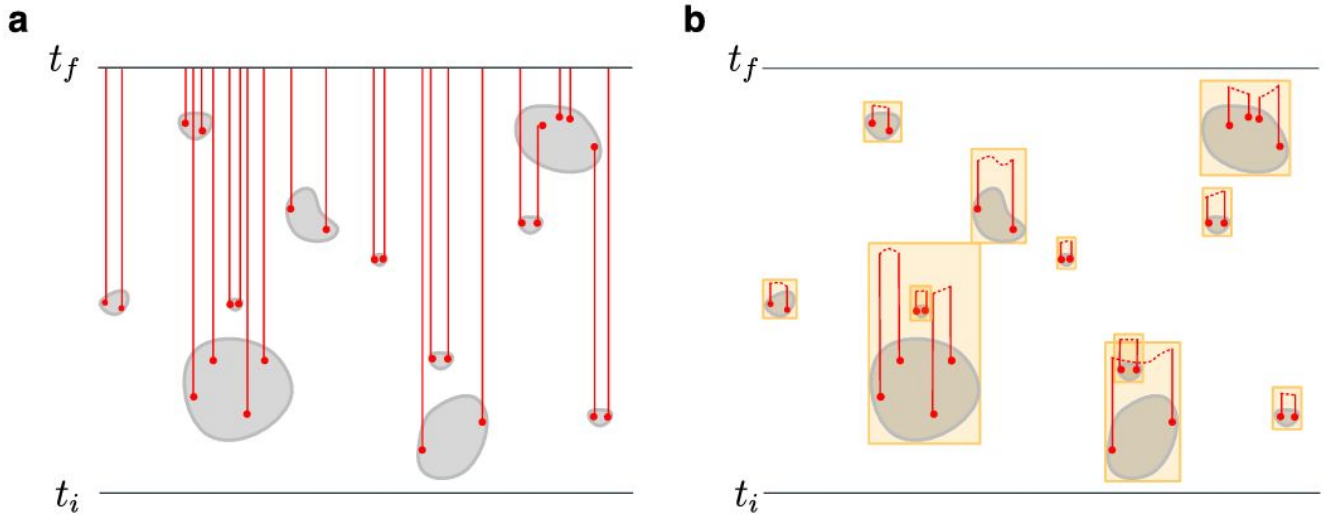


FIG. 15. (a) Schematic illustration of the worldlines of flux defects (red lines) created due to noise (contained in gray regions) in the circuit implementation of the non-Abelian TQD, not to scale. (b) The same spacetime history when operating with the just-in-time decoder, which pairs up the defects as soon as the correction can be deduced reliably. Each error cluster together with the associated flux worldlines is contained in some box (in yellow), and these boxes form a sparse distribution.

[1] Davydova et. al.

Just-in-Time Decoder

- Takeaway: we must correct X errors (flux defects) just-in-time!
- Flow:
  - Sample X and Z errors in the TQD circuit
  - Defer Z decoding, JIT-decode X and sample residual/twisted Z errors
  - Update Z decoding problem

Residual “Twisted” errors details in 2604.02033, 2609.19485

Simulating physical non-Cliffords

Simulation Procedure

- [Bravyi-König] Clifford circuits are classically simulatable (efficiently)
- Clifford gates map Paulis to other Paulis; thus can use Pauli frame / Stabilizer tableau to simulate.

$$\left(\begin{array}{ccc|ccc|c} x_{11} & \cdots & x_{1n} & z_{11} & \cdots & z_{1n} & r_1 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{n1} & \cdots & x_{nn} & z_{n1} & \cdots & z_{nn} & r_n \\ \hline x_{(n+1)1} & \cdots & x_{(n+1)n} & z_{(n+1)1} & \cdots & z_{(n+1)n} & r_{n+1} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{(2n)1} & \cdots & x_{(2n)n} & z_{(2n)1} & \cdots & z_{(2n)n} & r_{2n} \end{array}\right)$$

Simulation Procedure

- [Bravyi-König] Clifford circuits are classically simulatable (efficiently)
- Clifford gates map Paulis to other Paulis; thus can use Pauli frame / Stabilizer tableau to simulate.

  → Simple update rules

Pauli Stabilizer Update Rules

Starting with a state corresponding to +1 eigenstate of stabilizers generated by $\{G_j\}$,

1. Clifford gate C conjugates each generator G_j
2. When Pauli operator M is measured
   a. If M commutes with every stabilizer, measurement outcome deterministic and given by plus/minus product of all G_j eigenvalues
   b. If it anticommutes with at least one generator,

$$G_j \leftarrow G_j G_p. \tag{6}$$

The product $G_j G_p$ commutes with M because both G_j and G_p anticommute with M. Finally, replace the pivot G_p by M with the obtained random sign:

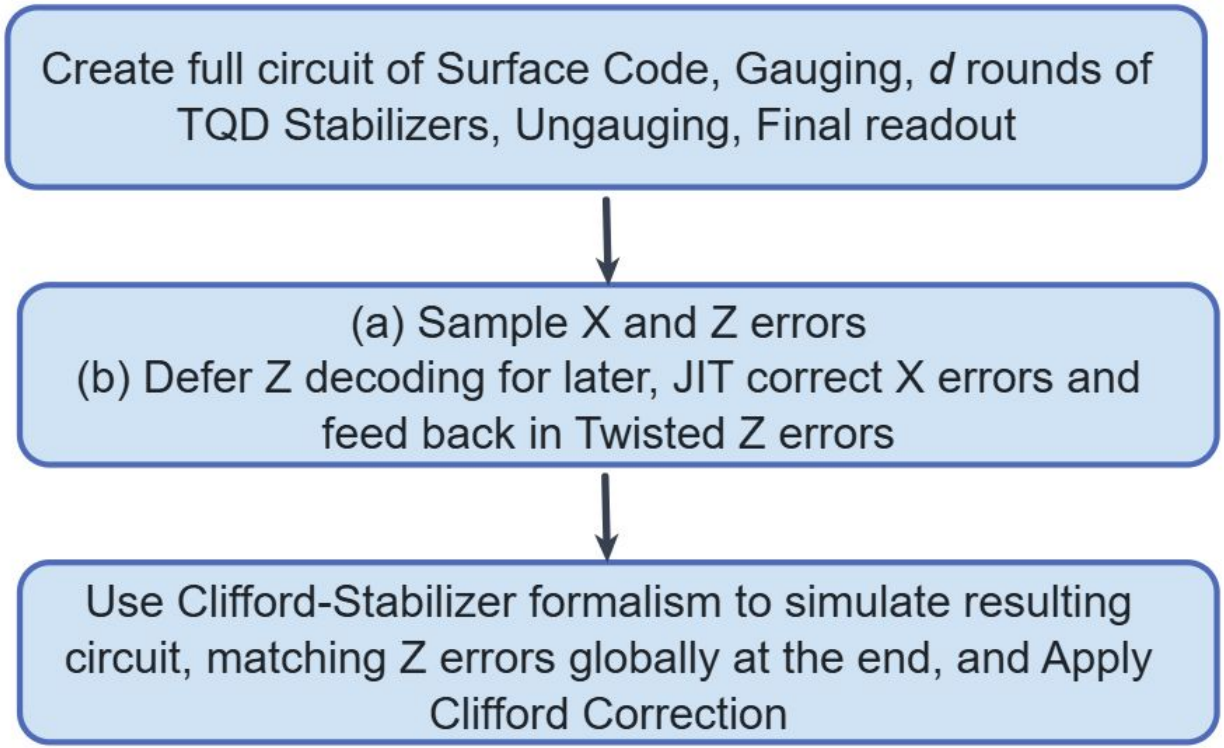
$$G_p \leftarrow (-1)^b M, \qquad b \in \{0, 1\}. \tag{7}$$

Clifford-Stabilizer Tableau

- For input noisy non-Clifford circuit, efficient algorithm to construct Clifford circuit with identical measurement outcome distribution!
- Idea: Can do similar tableau update when simulating Clifford Stabilizer group i.e. they effectively commute on some shared eigenspace

  - **Pauli dressings:** an error E maps G → DG, with D = [E, U] = EUE†U† a Pauli. Examples: XSX = iZS and X_i CZ_{ij} X_i = Z_j CZ_{ij}. All dressings are Z-type, so [M, G̃] ∈ ±⟨G^Z⟩.
  - **Update rules:** if M effectively anticommutes with some generator, the outcome is 50/50; with pivot G_p, update $G_j \leftarrow G_j\, G_p$ and $G_p \leftarrow \pm M$. Otherwise the outcome is deterministic and M = ±∏ G_j.
  - **Push errors:** Λ(U)·E = E·Λ(Ɗ_U(E))·Λ(U), with Ɗ_U(E) = E†UEU†. Only ancilla errors and X/Y data errors need pushing.
  - **Pauli proxy:** choose Ƥ_U such that [Ƥ_U, D] = [U, D] for all dressings D. Then Λ(U) → Λ(Ƥ_U), e.g. Ħ_XY → Ӿ, which removes all non-Clifford gates.

Simulation flow



References

[1]. Phys. Rev. X 16, 031001, *Universal fault tolerant quantum computation in 2D without getting tied in knots.* Margarita Davydova, Andreas Bauer, Julio C. Magdalena de la Fuente, Mark Webster, Dominic J. Williamson, and Benjamin J. Brown

[2]. Quantum 3, 205 (2019). *Magic State Distillation: Not as Costly as You Think*. Daniel Litinski.

[3]. *Magic State Cultivation: Growing T states as cheap as CNOT gates*. Craig Gidney, Noah Schutty, Cody Jones

[4]. *High-threshold decoding of non-Pauli codes for 2D universality*. Julio C. Magdalena de la Fuente, Noa Feldman, Jens Eisert, Andreas Bauer

[5]. *Efficiently estimating failure rates of fault-tolerant logical non-Clifford blocks*. Julio C. Magdalena de la Fuente, Thomas Scruby.

[6]. Exact efficient simulation of noisy logical magic states using Clifford stabilizers. Yugo Takada, Stephen D. Bartlett, Dominic J. Williamson.